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A NEW APPROACH TO TEXTURE ANALYSIS OF QUARK MASS MATRICES

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The triangular basis is proposed as an efficient way of analyzing general quark mass matrices. Applying the method to hermitian hierarchical matrices with five texture zeros, analytic predictions for quark mixing can be readily obtained. One unique texture pair is found most favorable with present data. Some remarks are also made concerning parallel textures between the up and down quark mass matrices with four zeros.

1 Mass matrices in the triangular basis

Texture analysis of quark mass matrices[1] has been a subject of interest for over two decades, and the advent of new data has rendered many popular textures into oblivion. A general and systematic approach based on the observed mass and mixing spectrum becomes both welcome and necessary. What we summarize here is one such approach using triangular matrices.

It is recently observed[2] that, as a result of the chiral nature of the electroweak force and the observed hierarchical structure among quark masses and mixing angles, the ten physical parameters of the quark mass matrices can be most simply encoded in hierarchical matrices of upper triangular form. For example, in the basis where M^U is diagonal, M^D is accurately given by,

$$M^U = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \quad M^D = \begin{pmatrix} m_d/V_{ud}^* & m_s V_{us} & m_b V_{ub} \\ 0 & m_s V_{cs} & m_b V_{cb} \\ 0 & 0 & m_b V_{tb} \end{pmatrix} \cdot (1+\mathcal{O}(\lambda^4)) \;, \quad (1)$$

where $\lambda = |V_{us}| = 0.22$. Note that in this form, the unphysical right-handed rotations have been eliminated to a good approximation.

Eq.(1) is actually one of ten[3] triangular pairs in the minimal-parameter basis (m.p.b.), where each matrix element consists of a simple product of a quark mass and certain CKM elements, and the CP-violating weak-phase is a linear combination of the phases of certain matrix elements. These ten pairs are simply related by weak basis transformation. Given any quark mass matrices, one can read off their physical content after converting them into one of the ten triangular pairs. Alternatively, one can start from upper triangular matrices and obtain mass matrices in other form, e.g. hermitian, and analyze texture zeros in the new basis.

2 Texture zeros of hermitian mass matrices

Based on the weak-scale quark mass relations $m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$ and $m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$, and on the hierarchical CKM mixings $V_{us} = \lambda$, $V_{cb} \sim \lambda^2$, $V_{ub} \sim \lambda^4$, and $V_{td} \sim \lambda^3$, we can write the properly normalized Yukawa matrices in the general hierarchical triangular form,

$$T^U = \begin{pmatrix} a_U\lambda^8 & b_U\lambda^6 & c_U\lambda^4 \\ 0 & d_U\lambda^4 & e_U\lambda^2 \\ 0 & 0 & 1 \end{pmatrix} , \qquad T^D = \begin{pmatrix} a_D\lambda^4 & b_D\lambda^3 & c_D\lambda^3 \\ 0 & d_D\lambda^2 & e_D\lambda^2 \\ 0 & 0 & 1 \end{pmatrix} . \tag{2}$$

Note that the diagonal elements are essentially the quark masses. The left-handed (LH) rotations are directly related to the off-diagonal elements, whose coefficients are of order one or much smaller to avoid fine-tuning[4] in getting the CKM mixing. This direct correspondence to masses and LH rotations is a unique feature of upper triangular matrices, and this feature makes the triangular matrices especially useful in analyzing quark mass matrices. As an example, we now turn to the analysis of texture zeros of hermitian quark mass matrices.

Starting from the triangular matrices of Eq. (2), we can easily write down their corresponding hermitian form:

$$Y^U = \begin{pmatrix} (a_U + c_U c_U^* + b_U b_U^*/d_U)\lambda^8 & (b_U + c_U e_U^*)\lambda^6 & c_U\lambda^4 \\ (b_U^* + c_U^* e_U)\lambda^6 & (d_U + e_U e_U^*)\lambda^4 & e_U\lambda^2 \\ c_U^*\lambda^4 & e_U^*\lambda^2 & 1 \end{pmatrix} \cdot (1 + \mathcal{O}(\lambda^4)) \tag{3}$$

$$Y^D = \begin{pmatrix} (a_D + b_D b_D^*/d_D)\lambda^4 & b_D\lambda^3 & c_D\lambda^3 \\ b_D^*\lambda^3 & d_D\lambda^2 & e_D\lambda^2 \\ c_D^*\lambda^3 & e_D^*\lambda^2 & 1 \end{pmatrix} \cdot (1 + \mathcal{O}(\lambda^2)) . \tag{4}$$

It is seen that diagonal zeros in the hermitian matrices imply definite relations between the diagonal and off-diagonal triangular parameters (i.e. masses and LH rotation angles). As a result of the different mass hierarchies in the up and down quark sector, there is a clear asymmetry between Eqs. (3) and (4) regarding their texture zeros. For example, the $(2,2)$ element can be zero for Y^U but not for Y^D; both the $(1,1)$ and $(1,2)$ elements can vanish with Y^U but not with Y^D.

Hermitian mass matrix textures can then be analyzed as follows. We first list all possible texture pairs $(M^U,\, M^D)$ directly from Eqs. (3) and (4). Each pair is then transformed into one of the ten triangular forms in the m.p.b. so that we can read-off possible relations between quark masses and mixing. Finally, the viability of each texture pair is tested by confronting its

predictions with data. In this way, we find no viable hermitian pairs with six zeros, including, *e.g.,* the Fritzsch texture[5]. We are thus lead to consider hermitian mass matrices with five texture zeros.

3 Hermitian mass matrices with 5 texture zeros

Following the procedure outlined above, we identify five candidates for viable hermitian pairs with five texture zeros, as was first obtained by Ramond, Roberts, and Ross (RRR) [6]. We now examine these five pairs analytically using the m.p.b. triangular matrices, and confront their predictions with data. The running quark mass values are taken from [7], and we use a recent update[8] on V_{ub}/V_{cb} and V_{td}/V_{ts}: $|V_{ub}/V_{cb}|_{\rm exp} = 0.093 \pm 0.014$, and $0.15 < |V_{td}/V_{ts}|_{\rm exp} < 0.24$. The results of this analysis [3] can be summarized as follows.

RRR patterns 1, 2, and 4 lead to the same predictions: $|V_{td}/V_{ts}| \simeq \sqrt{m_d/m_s} = 0.224 \pm 0.022$ and $|V_{ub}/V_{cb}| \simeq \sqrt{m_u/m_c} = 0.059 \pm 0.006$. Note that the numbers are independent of the scale at which the texture is valid, and that the latter prediction is on the low side and is disfavored by data.

The 5th RRR pattern also gives rise to two relations, each with two solutions depending on the sign of d_U. For the first relation,

$$|V_{us}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \sqrt{\frac{V_{cb}^2}{(m_c/m_t) \pm V_{cb}^2}} \right| , \tag{5}$$

where the relative phase is free as CP violation depends on additional phases in the mass matrices. Thus Eq. (5) is valid with a properly chosen phase. For the second relation,

$$\left| \frac{V_{ub}}{V_{cb}} \right| \simeq \sqrt{\frac{m_u}{m_c} \left(\frac{m_c}{m_t V_{cb}^2} \pm 1 \right)} = \begin{cases} 0.107 \pm 0.012 & (d_U > 0, \ \ 5a) \\ 0.068 \pm 0.011 & (d_U < 0, \ \ 5b) \end{cases} \tag{6}$$

where the numbers are given at M_Z with $V_{cb} = 0.040$. If the texture is valid at a much higher scale like $M_{\rm GUT}$, the weak scale predictions for the quark mixing will decrease by only a few percent. Comparing to data, we see that these predictions are marginally acceptable.

Finally, assuming the 3rd RRR texture to be valid at the weak scale, two predictions result: $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036 \pm 0.0004$, and $|V_{us}/V_{cs}| \simeq \sqrt{m_d/m_s} = 0.224 \pm 0.022$. Assuming the texture valid at the GUT scale will decrease the prediction for V_{ub} by a few percent. These predictions are in excellent agreement with data.

In summary, the 3rd RRR five-texture-zero pattern is currently most favorable.

4 Remarks on parallel textures

There are some recent studies[9] of 4-zero hermitian hierarchical mass matrices with parallel textures between M^U and M^D (i.e. M^U and M^D have the same texture zeros in the same locations). In particular, the generalized Fritzsch texture with zeros at (1,1), (1,3) and (3,1) has been a focus of interest. Using triangular matrices, one can easily see that generalized Fritzsch texture does not change the predictions for V_{ub}/V_{cb} and V_{td}/V_{ts} from its 5-zero hermitian counterparts (i.e. the 1st, 2nd, and 4th RRR patterns), and thus not favorable with data. In fact, no parallel hermitian hierarchical textures with 4 zeros is found to be favorable by current data[10]. In other words, the viable hermitian pairs display an asymmetry between the up and down textures. This asymmetry could serve as a useful guidline in building realistic models for quark-lepton masses.

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